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Perturbative matching of lattice and continuum heavy-light currents with NRQCD heavy quarks

Colin J. Morningstar
Dept. of Physics, University of California at San Diego, La Jolla, California 92093-0319

J. Shigemitsu
Physics Department, The Ohio State University, Columbus, OH 43210

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The temporal and spatial components of the heavy-light vector current and the spatial components of the axial current are expressed in terms of lattice-regulated operators suitable for simulations of B and D mesons. The currents are constructed by matching the appropriate scattering amplitudes in continuum QCD and a lattice model to one-loop order in perturbation theory. In the lattice theory, the heavy quarks are treated using the nonrelativistic (NRQCD) formulation and the light quarks are described by the tadpole-improved clover action. The light quarks are treated as massless. Our currents include relativistic and discretization corrections through $O(\alpha_s/M, a\,\alpha_s)$, where M is the heavy-quark mass, a is the lattice spacing, and α_s is the QCD coupling. As in our previous construction of the temporal component of the heavy-light axial current, mixing between several lattice operators is encountered at one-loop order, and $O(a\,\alpha_s)$ dimension-four improvement terms are identified.

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I. INTRODUCTION

An important goal of lattice gauge theory is to provide estimates of the hadronic matrix elements, such as those of the electroweak currents and effective four-fermion operators, which are needed for precision tests of the standard model. Numerical simulations of quarks and gluons in lattice-regulated QCD currently provide the only means of calculating such matrix elements from first principles taking long-distance QCD dynamics fully into account. An important ingredient in these calculations is the construction of the lattice current operators which match the currents defined in the continuum to some desired accuracy. In this paper, we express the heavy-light vector and axial-vector currents in terms of operators defined in a lattice model in which the heavy quarks are treated using the nonrelativistic (NRQCD) formulation [1], the light quarks are described by the tadpole-improved clover action [2], and the gluons are governed by the standard Wilson action. These heavy-light currents are important in studies of heavy meson leptonic and semileptonic decays (for recent reviews, see Refs. [3–5]). The lattice currents are determined by matching scattering amplitudes in the lattice model to those in continuum QCD to one-loop order in perturbation theory. The matching is carried out through $O(1/M, \alpha_s/M, a\,\alpha_s)$ where M is the heavy quark mass, a is the lattice spacing, and α_s is the QCD coupling. The light quarks are treated as massless.

Each heavy-light current J_μ, defined in some continuum renormalization scheme, can be written

$$J_\mu = \sum_j C_j^J \; J^{(j)}_{\mu,\mathrm{lat}} + O(\alpha_s^2, a^2, 1/M_0^2, \alpha_s a/M_0), \qquad (1)$$

where $J^{(j)}_{\mu,\mathrm{lat}}$ are operators defined in the effective lattice theory, M_0 is the heavy quark (bare) mass parameter appearing in the lattice NRQCD action, and the C_j^J coefficients are c-numbers which depend only on α_s and aM_0. The goal is to identify the operators $J^{(j)}_{\mu,\mathrm{lat}}$ and to calculate the matching coefficients $C_j^J(\alpha_s, aM_0)$. The procedure for doing this was described in a previous article [6] in which the temporal component of the axial-vector current was studied. For convenience, we reiterate the salient steps: (1) select a quark-gluon scattering process induced by the heavy-light current of interest and calculate the one-loop amplitude for this process in continuum QCD; (2) expand the amplitude in powers of $1/M$; (3) identify operators in the lattice theory, usually by inspection, which reproduce the terms in this expansion; (4) calculate the one-loop mixing matrix of these operators in the lattice theory; and (5) adjust the C_j^J coefficients to produce a linear combination of lattice current operators whose one-loop scattering amplitude agrees with that determined in step 2 to a given order in $1/M$ and a. In this paper, we apply the above procedure to the spatial components A_k of the axial-vector current and the spatial and temporal components of the vector current

V_μ. We omit much of the calculational details since they are similar to those already described in Ref. [6].

In Sec. II, we start from a continuum QCD calculation and identify the current operators in the effective theory needed to reproduce the continuum current through $O(\alpha_s/M)$. In Sec. III, we describe the lattice theory and the one-loop mixing calculation among the $J^{(j)}_{\mu,\rm lat}$. The matching procedure is then completed in Sec. IV. In Sec. V, we discuss $O(a\,\alpha_s)$ corrections in the static limit, and issues pertaining to terms that behave as $\alpha_s \log(aM)$ are dealt with in Sec. VI. The paper concludes with a summary of our results in Sec. VII.

II. OPERATOR IDENTIFICATION

The heavy-light vector and axial-vector currents are given as usual by $V_\mu(x) = \bar{q}(x)\,\gamma_\mu\,h(x)$ and $A_\mu(x) = \bar{q}(x)\,\gamma_5\gamma_\mu\,h(x)$, respectively, where $q(x)$ is the light quark field, $h(x)$ is the heavy quark field, and γ_μ are the standard Dirac γ-matrices in Euclidean space-time which satisfy $\{\gamma_\mu,\gamma_\nu\} = 2\,\delta_{\mu\nu}$, and $\gamma_5 = \gamma_0\gamma_1\gamma_2\gamma_3$. Euclidean-space four-vectors are defined in terms of Minkowski-space four-vectors (indicated by an underline) using $x_0 = i\,\underline{x}^0$ and $x_j = \underline{x}^j = -\underline{x}_j$, for $j=1,2,3$. For the derivative operator, $\partial_0 = -i\underline{\partial}_0$ and $\partial_j = \underline{\partial}_j = -\underline{\partial}^j$. The currents are related by $J_0 = \underline{J}_0$, and $J^j = -i\,\underline{J}^j$. For further details, see Refs. [6,7].

The first step in identifying the necessary operators in the effective lattice theory is to calculate the matrix elements of the continuum QCD currents for an incoming heavy quark of momentum p and an outgoing light quark of momentum p'. Using the on-shell mass and wave function renormalization scheme in Feynman gauge and expanding in $1/M$ (except for $u_h(p)$), one finds to one-loop order in perturbation theory

$$\langle\, q(p')\,|\,J_\mu\,|\,h(p)\,\rangle_{QCD} = \bar{u}_q(p')\;W^J_\mu(p',p)\;u_h(p), \qquad (2)$$

where

$$\begin{aligned} W^J_\mu(p',p) = {} & a_1\,\Gamma^J_\mu - a_2\,\frac{ip_\mu}{M}\,\Gamma^J_0\gamma_0 - a_3\,\frac{p\cdot p'}{M^2}\,\Gamma^J_\mu \\ & - a_4\,\frac{ip'_\mu}{M}\,\Gamma^J_0\gamma_0 + a_5\,\frac{p\cdot p'}{M^2}\,\frac{ip_\mu}{M}\,\Gamma^J_0\gamma_0 \\ & + \quad O(1/M^2), \end{aligned} \qquad (3)$$

with

$$\begin{aligned} a_1 = {} & 1 + \frac{\alpha_s}{3\pi}\left[3\,\ln\frac{M}{\lambda} - \frac{11}{4}\right], \\ a_2 = {} & \frac{\alpha_s}{3\pi}\;2, \\ a_3 = {} & \frac{\alpha_s}{3\pi}\left[\;6\,\ln\frac{M}{\lambda} - \frac{8\pi}{3}\,\frac{M}{\lambda} + \frac{1}{2}\right], \\ a_4 = {} & \frac{\alpha_s}{3\pi}\left[-2\,\ln\frac{M}{\lambda} + \frac{1}{2}\right], \\ a_5 = {} & \frac{\alpha_s}{3\pi}\left[-4\,\ln\frac{M}{\lambda} + 5\right]. \end{aligned} \qquad (4)$$

For the vector current, $\Gamma^V_\mu = \gamma_\mu$ and for the axial-vector current, $\Gamma^A_\mu = \gamma_5\gamma_\mu$. $u_h(p)$ and $u_q(p')$ are the standard spinors for the heavy and light quarks, respectively, which satisfy the Dirac equation. The light quark mass is set equal to zero. Ultraviolet divergences are treated using dimensional regularization, and fully anti-commuting γ_5 matrices are used. We use a gluon mass λ to regulate infrared divergences. Note that M is the heavy-quark pole mass.

In lattice NRQCD, the heavy quark is described in terms of a two-component (in spin space) field $\psi(x)$. The Dirac field $h(x)$ is related to $\psi(x)$ (and the antiquark field $\tilde{\psi}(x)$) by a unitary Foldy-Wouthuysen transformation [8],

$$h(x) = U^{-1}_{FW}\;\begin{pmatrix}\psi(x)\\ \tilde{\psi}(x)\end{pmatrix}. \qquad (5)$$

This transformation decouples the upper and lower components of the Dirac field, thereby separating the quark field from the antiquark field. To facilitate the identification of lattice NRQCD operators capable of matching Eq. (2), we similarly transform the external state spinor $u_h(p)$ into a nonrelativistic Pauli spinor:

$$u_h(p) = \left[1 - \frac{i}{2M}\;\boldsymbol{\gamma}\cdot\mathbf{p}\right]\,u_Q(p) + O(1/M^2), \qquad (6)$$

where

$$u_Q(p) = \begin{pmatrix} U_Q \\ 0 \end{pmatrix}, \qquad (7)$$

and U_Q is a two-component external state spinor depending only on the spin of the heavy quark. Note that we are working in the Dirac-Pauli representation. Using Eq. (6), the relation $\gamma_0\,u_Q(p) = u_Q(p)$, and the Dirac equation for the light quark $\bar{u}_q(p')p'_0 = -\bar{u}_q(p')\boldsymbol{\gamma}\cdot\mathbf{p}'\gamma_0$, the spatial components of Eq. (2) may be written

$$\begin{aligned} \langle\, q(p')\,|\,J_k\,|\,h(p)\,\rangle_{QCD} = {} & \eta_0\Omega^{(0)}_k + \eta_1\Omega^{(1)}_k + \eta_2\Omega^{(2)}_k \\ & + \eta_3\Omega^{(3)}_k + \eta_4\Omega^{(4)}_k \\ & + O(\alpha_s^2, 1/M^2), \end{aligned} \qquad (8)$$

with

$$\Omega^{(0)}_k = \bar{u}_q(p')\;\Gamma^J_k\;u_Q(p), \qquad (9)$$

$$\Omega^{(1)}_k = -i\bar{u}_q(p')\;\Gamma^J_k\,\frac{\boldsymbol{\gamma}\cdot\mathbf{p}}{2M}\;u_Q(p), \qquad (10)$$

$$\Omega^{(2)}_k = i\bar{u}_q(p')\;\frac{\boldsymbol{\gamma}\cdot\mathbf{p}'}{2M}\gamma_0\;\Gamma^J_k\;u_Q(p), \qquad (11)$$

$$\Omega^{(3)}_k = -i\bar{u}_q(p')\,\frac{p_k}{2M}\;\Gamma^J_0\;u_Q(p), \qquad (12)$$

$$\Omega^{(4)}_k = -i\bar{u}_q(p')\;\frac{p'_k}{2M}\;\Gamma^J_0\;u_Q(p). \qquad (13)$$

The coefficients in Eq. (8) are given by

$$\begin{aligned}
\eta_0 &= \quad a_1 \;=\; 1+\alpha_s\;\tilde{B}_0, \\
\eta_1 &= \quad \eta_0, \\
\eta_2 &= 2\,a_3 \;=\alpha_s\;\tilde{B}_2, \\
\eta_3 &= 2\,a_2 \;=\alpha_s\;\tilde{B}_3, \\
\eta_4 &= 2\,a_4 \;=\alpha_s\;\tilde{B}_4,
\end{aligned} \tag{14}$$

where

$$\begin{aligned}
\tilde{B}_0 &= \frac{1}{3\pi}\left[3\,\ln\frac{M}{\lambda}-\frac{11}{4}\right], \\
\tilde{B}_2 &= \frac{1}{3\pi}\left[12\,\ln\frac{M}{\lambda}-\frac{16\pi}{3}\frac{M}{\lambda}+1\right], \\
\tilde{B}_3 &= \frac{1}{3\pi}4, \\
\tilde{B}_4 &= \frac{1}{3\pi}\left[-4\,\ln\frac{M}{\lambda}+1\right].
\end{aligned} \tag{15}$$

Having obtained the $1/M$ expansion of the above scattering amplitude in continuum QCD, the next step is to identify operators in the lattice effective theory that can reproduce the terms in this expansion. An inspection of Eq. (8) suggests immediately that matrix elements of the following five lattice operators should be considered:

$$J^{(0)}_{k,\mathrm{lat}}(x) = \bar{q}(x)\,\Gamma^J_k\,Q(x), \tag{16}$$

$$J^{(1)}_{k,\mathrm{lat}}(x) = \frac{-1}{2M_0}\bar{q}(x)\,\Gamma^J_k\,\boldsymbol{\gamma\cdot\nabla}\,Q(x), \tag{17}$$

$$J^{(2)}_{k,\mathrm{lat}}(x) = \frac{-1}{2M_0}\bar{q}(x)\,\boldsymbol{\gamma\cdot}\overleftarrow{\boldsymbol{\nabla}}\,\gamma_0\;\Gamma^J_k\,Q(x), \tag{18}$$

$$J^{(3)}_{k,\mathrm{lat}}(x) = \frac{-1}{2M_0}\bar{q}(x)\,\Gamma^J_0\nabla_k\,Q(x) \tag{19}$$

$$J^{(4)}_{k,\mathrm{lat}}(x) = \frac{1}{2M_0}\bar{q}(x)\,\overleftarrow{\nabla}_k\Gamma^J_0\,Q(x), \tag{20}$$

where $q(x)$ is now the light quark field in the lattice theory, M_0 is the bare heavy quark mass, and $Q(x)$ is related to the heavy quark field $\psi(x)$ in lattice NRQCD by

$$Q(x) = \begin{pmatrix}\psi(x)\\ 0\end{pmatrix}. \tag{21}$$

We use the bare quark mass M_0 in the above definitions since it is the natural choice to use in lattice simulations and because the pole mass M is not well defined beyond perturbation theory. Definitions of the lattice derivatives are given in the next section.

For the temporal components of the currents $J_0 = V_0$ or A_0, the leading $1/M$ behavior is given by

$$\begin{aligned}
\langle\, q(p')\,|\,J_0\,|\,h(p)\,\rangle_{QCD} &= \eta^t_0\;\Omega^{(0)}_0 + \eta^t_1\;\Omega^{(1)}_0 + \eta^t_2\;\Omega^{(2)}_0 \\
&\quad + O(\alpha_s^2, 1/M^2),
\end{aligned} \tag{22}$$

with

$$\Omega^{(0)}_0 = \bar{u}_q(p')\;\Gamma^J_0\;u_Q(p), \tag{23}$$

$$\Omega^{(1)}_0 = -i\bar{u}_q(p')\;\Gamma^J_0\frac{\boldsymbol{\gamma\cdot}\mathbf{p}}{2M}\;u_Q(p), \tag{24}$$

$$\Omega^{(2)}_0 = i\bar{u}_q(p')\;\frac{\boldsymbol{\gamma\cdot}\mathbf{p'}}{2M}\gamma_0\;\Gamma^J_0\;u_Q(p). \tag{25}$$

The coefficients in Eq. (22) are given by

$$\begin{aligned}
\eta^t_0 &= \quad (a_1+a_2) \;=\; 1+\alpha_s\;\tilde{B}^t_0, \\
\eta^t_1 &= \quad (a_1-a_2) \;=\; 1+\alpha_s\;\tilde{B}^t_1, \\
\eta^t_2 &= 2\,(a_3+a_4+a_5) \;=\alpha_s\;\tilde{B}^t_2,
\end{aligned} \tag{26}$$

where

$$\begin{aligned}
\tilde{B}^t_0 &= \frac{1}{3\pi}\left[3\,\ln\frac{M}{\lambda}-\frac{3}{4}\right], \\
\tilde{B}^t_1 &= \frac{1}{3\pi}\left[3\,\ln\frac{M}{\lambda}-\frac{19}{4}\right], \\
\tilde{B}^t_2 &= \frac{1}{3\pi}\left[12-\frac{16\pi}{3}\frac{M}{\lambda}\right].
\end{aligned} \tag{27}$$

Again, it is straightforward to identify operators in the lattice effective theory which can reproduce these terms:

$$J^{(0)}_{0,\mathrm{lat}}(x) = \bar{q}(x)\,\Gamma^J_0\,Q(x), \tag{28}$$

$$J^{(1)}_{0,\mathrm{lat}}(x) = \frac{-1}{2M_0}\bar{q}(x)\,\Gamma^J_0\,\boldsymbol{\gamma\cdot\nabla}\,Q(x), \tag{29}$$

$$J^{(2)}_{0,\mathrm{lat}}(x) = \frac{-1}{2M_0}\bar{q}(x)\,\boldsymbol{\gamma\cdot}\overleftarrow{\boldsymbol{\nabla}}\,\gamma_0\;\Gamma^J_0\,Q(x). \tag{30}$$

These operators were previously used in the expansion of the temporal component of the axial-vector heavy-light current in Refs. [6,9].

III. MIXING MATRIX

Having identified the necessary operators in the lattice-regulated effective field theory, we next need to calculate the mixing matrix Z^J_{ij} of these operators defined by

$$\begin{aligned}
\langle\, q(p')\,|\,J^{(i)}_{\mu,\mathrm{lat}}\,|\,h(p)\,\rangle_{\mathrm{lat}} &= \sum_j\; Z^J_{ij}\;\Omega^{(j)}_\mu \\
&\quad + O(\alpha_s^2, 1/M^2, a^2, \alpha_s a/M),
\end{aligned} \tag{31}$$

where $\Omega^{(j)}_\mu$ are the five $\Omega^{(j)}_k$ defined in Eqs. (9)-(13) for $\mu = k = 1,2,3$ and the three $\Omega^{(j)}_0$ defined in Eqs. (23)-(25) for $\mu = 0$. The determination of the Z^J_{ij} factors is the most difficult aspect of the current construction. These factors are determined numerically using lattice perturbation theory. The same methods used in Ref. [6] are applied here. We also use the same lattice action as in Ref. [6], but for the convenience of the reader, we

restate these actions below. The Feynman rules for our heavy and light quark lattice action and further details of lattice perturbation theory are given in Refs. [6,7].

For the heavy quark, the following NRQCD action density on the lattice [1] is used:

$$
\begin{aligned}
a\mathcal{L}_{NRQCD} = \psi^\dagger(x)\ \psi(x) \\
&-\psi^\dagger(x+a_t)\left(1-\frac{a\delta H}{2}\right)\left(1-\frac{aH_0}{2n}\right)^n \frac{U_4^\dagger(x)}{u_0} \\
&\times\left(1-\frac{aH_0}{2n}\right)^n\left(1-\frac{a\delta H}{2}\right)\psi(x),
\end{aligned}
\tag{32}
$$

where

$$H_0 = -\frac{\Delta^{(2)}}{2M_0}, \tag{33}$$

$$\delta H = -c_B\,\frac{g}{2M_0}\,\boldsymbol{\sigma}\cdot\mathbf{B}, \tag{34}$$

$U_\mu(x)$ are standard gauge-field link variables, n is an integer, σ_j are the Pauli matrices, u_0 is the mean link parameter introduced by the tadpole improvement procedure [10], and the QCD coupling g is related to α_s by $\alpha_s = g^2/(4\pi)$. At tree level, $c_B = 1$; the one-loop contribution to c_B is an $O(\alpha^2)$ effect in our mixing matrix calculation and hence can be ignored here. The chromomagnetic field is given by $B_j(x) = -\frac{1}{2}\epsilon_{jlm}F_{lm}(x)$, where the Hermitian and traceless field strength tensor $F_{\mu\nu}(x)$ is defined at the sites of the lattice in terms of clover-leaf operators:

$$
\begin{aligned}
F_{\mu\nu}(x) &= \mathcal{F}_{\mu\nu}(x) - \frac{1}{3}\mathrm{Tr}\mathcal{F}_{\mu\nu}(x), \\
\mathcal{F}_{\mu\nu}(x) &= \frac{-i}{2a^2g}\left(\Omega_{\mu\nu}(x) - \Omega^\dagger_{\mu\nu}(x)\right), \\
\Omega_{\mu\nu}(x) &= \frac{1}{4u_0^4}\sum_{\{(\alpha,\beta)\}_{\mu\nu}} U_\alpha(x)U_\beta(x+a_\alpha) \\
&\quad\times U_{-\alpha}(x+a_\alpha+a_\beta)U_{-\beta}(x+a_\beta),
\end{aligned}
\tag{35}
$$

with $\{(\alpha,\beta)\}_{\mu\nu} = \{(\mu,\nu),(\nu,-\mu),(-\mu,-\nu),(-\nu,\mu)\}$ for $\mu \neq \nu$. The lattice derivatives in our action and in the current operators are given by

$$
\begin{aligned}
a\nabla_\mu O(x) &= \frac{1}{2u_0}[U_\mu(x)O(x+a_\mu) \\
&\quad -U^\dagger_\mu(x-a_\mu)O(x-a_\mu)],
\end{aligned}
\tag{36}
$$

$$
\begin{aligned}
O(x)\ a\overleftarrow{\nabla}_\mu &= \frac{1}{2u_0}[O(x+a_\mu)U^\dagger_\mu(x) \\
&\quad -O(x-a_\mu)U_\mu(x-a_\mu)],
\end{aligned}
\tag{37}
$$

$$
\begin{aligned}
a^2\Delta^{(2)}O(x) &= \sum_{k=1}^{3}(u_0^{-1}[U_k(x)O(x+a_k) \\
&\quad +U^\dagger_k(x-a_k)O(x-a_k)] - 2O(x)),
\end{aligned}
\tag{38}
$$

$$
\begin{aligned}
a^2\nabla^{(2)}O(x) &= \sum_{\mu=0}^{3}(u_0^{-1}[U_\mu(x)O(x+a_\mu) \\
&\quad +U^\dagger_\mu(x-a_\mu)O(x-a_\mu)] - 2O(x)),
\end{aligned}
\tag{39}
$$

where $O(x)$ is an operator defined at lattice site x with appropriate color structure.

For the light quarks, we use the clover action [2],

$$
\begin{aligned}
a\mathcal{L}_{light} &= \overline{q}\,\not{\nabla} q - a\,\frac{r}{2}\,\overline{q}\,\nabla^{(2)}q + m_0\,\overline{q}q \\
&\quad -iga\frac{r}{4}\sum_{\mu,\nu}\ \overline{q}\,\sigma_{\mu\nu}F_{\mu\nu}\,q,
\end{aligned}
\tag{40}
$$

where $\not{\nabla} = \sum_\mu \gamma_\mu\nabla_\mu$, m_0 is the bare light quark mass, $\sigma_{\mu\nu} = \frac{1}{2}[\gamma_\mu,\gamma_\nu]$, and we set the Wilson parameter $r=1$. The one-loop correction to the clover coefficient is an $O(\alpha_s^2)$ effect in our matching calculation and is neglected here.

At one-loop order, the mixing matrix elements may be written [11]

$$Z^J_{ij} = \delta_{ij} + \alpha_s\left\{\left[\tfrac{1}{2}(\tilde{C}_q+\tilde{C}_Q)+\tilde{C}_m(1-\delta_{i0})\right]\delta_{ij}+\tilde{\zeta}^J_{ij}\right\}, \tag{41}$$

where $\tilde{C}_q$ and $\tilde{C}_Q$ are the contributions from the light- and heavy-quark external leg corrections (that is, from wave function renormalization factors), and $\tilde{\zeta}^J_{ij}$ denote the contributions from the vertex corrections. Our use of an on-shell renormalization scheme with lattice operators defined in terms of the bare mass M_0 is responsible for the term proportional to $\tilde{C}_m$, where

$$M = [1+\alpha_s\,\tilde{C}_m]\,M_0 + O(\alpha_s^2). \tag{42}$$

Note that although the current operators $J^{(i)}_{\mu,\mathrm{lat}}$ are defined using M_0, the pole mass M appears in $\Omega^{(j)}_\mu$. Within the context of finite-order perturbation theory, the pole mass is an observable quantity. It is gauge invariant and regularization scheme independent. Hence, it is the natural heavy-quark mass definition to use when matching amplitudes in different schemes. However, in reality, a confined quark has no pole mass. Once the matching coefficients C^J_j are obtained in terms of aM, it is then necessary to eliminate any dependence on M in favor of some quark mass which is defined beyond perturbation theory. Clearly, the mass parameter M_0 appearing in the lattice action is the most suitable and convenient choice. The relationship between M and M_0 given in Eq. (42) is then used to re-express the C^J_j coefficients in terms of M_0.

The factors in Eq. (41) may be further decomposed:

$$
\begin{aligned}
\tilde{C}_q &= C_q + \frac{2}{3\pi}\ln a\lambda + C^{\mathrm{TI}}_q, \\
\tilde{C}_Q &= C_Q - \frac{4}{3\pi}\ln a\lambda, \\
\tilde{C}_m &= C_m + C^{\mathrm{TI}}_m, \\
\tilde{\zeta}^J_{ij} &= \zeta^J_{ij} + \zeta^{J,\mathrm{TI}}_{ij} + \zeta^{J,\mathrm{IR}}_{ij},
\end{aligned}
\tag{43}
$$

where C_q, C_Q, C_m, and ζ_{ij}^J are infrared finite and independent of the tadpole improvement factor u_0, and $\zeta_{ij}^{J,\mathrm{IR}}$ and $\zeta_{ij}^{J,\mathrm{TI}}$ contain the infrared divergences and tadpole improvement contributions, respectively, from the vertex corrections. Contributions to C_q and C_m from the tadpole improvement counterterms are denoted by C_q^{TI} and C_m^{TI}, respectively. The tadpole improvement terms are

$$\begin{aligned} C_q^{\mathrm{TI}} &= -u_0^{(2)}, \\ C_m^{\mathrm{TI}} &= -u_0^{(2)} \left(1 - \frac{3}{2\,n\,aM_0}\right), \\ \zeta_{ij}^{J,\mathrm{TI}} &= u_0^{(2)} \delta_{ij}(1-\delta_{i0}), \end{aligned} \tag{44}$$

where $u_0 = 1 - \alpha_s u_0^{(2)} + O(\alpha_s^2)$. For the usual plaquette definition $u_0 = \langle \frac{1}{3}\mathrm{Tr}U_\Box\rangle^{1/4}$ in the Wilson gluonic action, $u_0^{(2)} = \pi/3$. For massless clover quarks, $C_q = 1.030$. Values for C_Q and C_m are given in Ref. [6].

IV. MATCHING

To complete the operator construction for J_k, we invert Eq. (31) to get $\Omega_\mu^{(j)} = \sum_i (Z^J)^{-1}_{ji} \langle J_{\mu,\mathrm{lat}}^{(i)}\rangle_{\mathrm{lat}}$, substitute this into Eq. (8), peel off the dependence on the external states, and use Eqs. (14), (15), (41), (42), and (43) to obtain

$$\begin{aligned} J_k = & \left(1 + \alpha_s\left[B_0 - C_{Qq} - \tau_0^{J_k}\right]\right) J_{k,\mathrm{lat}}^{(0)} \\ & + \left(1 + \alpha_s\left[B_1 - C_{Qqm} - \tau_1^{J_k} - \tau_1^{\mathrm{TI}}\right]\right) J_{k,\mathrm{lat}}^{(1)} \\ & + \alpha_s\left[B_2 - \tau_2^{J_k}\right] J_{k,\mathrm{lat}}^{(2)} \\ & + \alpha_s\left[B_3 - \tau_3^{J_k}\right] J_{k,\mathrm{lat}}^{(3)} \\ & + \alpha_s\left[B_4 - \tau_4^{J_k}\right] J_{k,\mathrm{lat}}^{(4)} \\ & + O(\alpha_s^2, a^2, 1/M^2, \alpha_s a/M), \end{aligned} \tag{45}$$

where $\tau_j^{J_k} = \zeta_{0j}^{J_k} + \zeta_{1j}^{J_k}$, $\tau_1^{\mathrm{TI}} = u_0^{(2)}$,

$$\begin{aligned} B_0 &= \frac{1}{\pi}\left[\ln(aM_0) - \frac{11}{12}\right] = B_1, \\ B_2 &= \frac{1}{\pi}\left[4\,\ln(aM_0) + \frac{1}{3}\right], \\ B_3 &= \frac{4}{3\pi}, \\ B_4 &= \frac{1}{\pi}\left[-\frac{4}{3}\ln(aM_0) + \frac{1}{3}\right], \end{aligned} \tag{46}$$

and

$$C_{Qq} = \frac{1}{2}(C_q + C_q^{\mathrm{TI}} + C_Q), \tag{47}$$

$$C_{Qqm} = C_{Qq} + C_m + C_m^{\mathrm{TI}}. \tag{48}$$

As expected, the infrared divergences from the various terms in the expansion coefficients cancel. Results for $\tau_j^{V_k}$ and $\tau_j^{A_k}$ for various values of aM_0 are listed in Tables I and II, respectively.

For the temporal component of the vector current, we substitute the inverted Eq. (31) into Eq. (22), remove the dependence on the external states, and use Eqs. (26), (27), (41), (42), and (43) to obtain

$$\begin{aligned} V_0 = & \left(1 + \alpha_s\left[B_0^t - C_{Qq} - \tau_0^{V_0}\right]\right) J_{0,\mathrm{lat}}^{(0)} \\ & + \left(1 + \alpha_s\left[B_1^t - C_{Qqm} - \tau_1^{V_0} - \tau_1^{\mathrm{TI}}\right]\right) J_{0,\mathrm{lat}}^{(1)} \\ & + \alpha_s\left[B_2^t - \tau_2^{V_0}\right] J_{0,\mathrm{lat}}^{(2)} \\ & + O(\alpha_s^2, a^2, 1/M^2, \alpha_s a/M), \end{aligned} \tag{49}$$

where

$$\begin{aligned} B_0^t &= \frac{1}{\pi}\left[\ln(aM_0) - \frac{1}{4}\right], \\ B_1^t &= \frac{1}{\pi}\left[\ln(aM_0) - \frac{19}{12}\right], \\ B_2^t &= \frac{4}{\pi}. \end{aligned} \tag{50}$$

Again, all infrared divergences cancel. Results for $\tau_j^{V_0}$ for various aM_0 values are given in Table III.

From Eqs. (45) and (49), one easily obtains the matching coefficients C_j^J of Eq. (1) which expresses the heavy-light currents in terms of operators suitable for numerical simulations using lattice NRQCD. These coefficients can be written in the form

$$C_i^J(\alpha_s, aM_0) = \begin{cases} 1 + \alpha_s\ \rho_i^J(aM_0) + O(\alpha_s^2), & (i = 0,1), \\ \alpha_s\ \rho_i^J(aM_0) + O(\alpha_s^2), & (i \geq 2). \end{cases} \tag{51}$$

The values for $\rho_i^{V_k}$, $\rho_i^{A_k}$, and $\rho_i^{V_0}$ for various values of aM_0 are listed in Tables IV, V, and VI, respectively.

V. $O(a\alpha_S)$ CORRECTIONS

In this section, discretization corrections in the static limit $aM_0 \to \infty$ are discussed. To study such corrections, we calculated all τ_i^J for several large values of the heavy quark mass. With the exception of τ_2^J, all of the τ_i^J tended tamely to finite values in the static limit. However, as aM_0 became large, we found that the magnitude of τ_2^J began to grow linearly with aM_0:

$$\tau_2^J \xrightarrow{aM_0 \gg 1} -2aM_0\ \zeta_{disc}^J + \mathrm{constant}, \tag{52}$$

where the ζ^J_{disc} are constants independent of aM_0. We determined the ζ^J_{disc} by computing τ^J_2 for $aM_0 = 10.0, 25.0, 100.0, 400.0, 1000.0$ and 5000.0. We then fit the results for $-\tau^J_2/(2aM_0)$ to a quadratic polynomial in $1/(aM_0)$ and obtained the following limiting values:

$$\zeta^{A_0}_{disc} = \quad 1.029(2), \tag{53}$$
$$\zeta^{V_k}_{disc} = \quad 1.031(1), \tag{54}$$
$$\zeta^{A_k}_{disc} = -0.063(1), \tag{55}$$
$$\zeta^{V_0}_{disc} = -0.063(1). \tag{56}$$

The discretization factor $\zeta^{A_0}_{disc}$ for the temporal component of the axial-vector current was previously calculated in Ref. [6], but its determination was not done very accurately. We have recalculated $\zeta^{A_0}_{disc}$ here to a much higher precision.

Since the contributions proportional to ζ^J_{disc} are purely $O(a\alpha_s)$ discretization corrections, one may absorb these terms into the lattice current operators. Improved current operators can be defined using

$$\hat{J}^{(0)}_{\mu,\mathrm{lat}}(x) = J^{(0)}_{\mu,\mathrm{lat}}(x) + \alpha_s\, \zeta^J_{disc}\, J^{(disc)}_{\mu,\mathrm{lat}}(x), \tag{57}$$
$$\hat{J}^{(j)}_{\mu,\mathrm{lat}}(x) = J^{(j)}_{\mu,\mathrm{lat}}(x), \quad (j > 0), \tag{58}$$

where

$$\begin{aligned} J^{(disc)}_{\mu,\mathrm{lat}}(x) &= 2aM_0\, J^{(2)}_{\mu,\mathrm{lat}}(x), \\ &= -a\, \bar{q}(x)\, \boldsymbol{\gamma}\cdot \overleftarrow{\boldsymbol{\nabla}}\, \gamma_0 \Gamma^J_\mu\, Q(x). \end{aligned} \tag{59}$$

The mixing of the improved operators is given by

$$\langle\, q(p')\,|\, \hat{J}^{(i)}_{\mu,\mathrm{lat}}\,|\, h(p)\,\rangle_{\mathrm{lat}} = \sum_j\, \hat{Z}^J_{ij}\, \Omega^{(j)}_\mu + O(\alpha_s^2, 1/M^2, a^2, \alpha_s a/M), \tag{60}$$

where

$$\hat{Z}^J_{0i} = Z^J_{0i} + 2\alpha_s\, aM_0 \zeta^J_{disc}\, \delta_{2i}, \tag{61}$$
$$\hat{Z}^J_{ji} = Z^J_{ji}, \quad (j > 0). \tag{62}$$

Then the coefficients $\hat{C}^J_j$ of these operators are

$$\hat{C}^J_2 = C^J_2 - 2aM_0\, \zeta^J_{disc}\, \alpha_s, \tag{63}$$
$$\hat{C}^J_i = C^J_i, \quad (i \neq 2), \tag{64}$$

and the heavy-light currents are given by

$$J_\mu = \sum_j \hat{C}^J_j\, \hat{J}^{(j)}_{\mu,\mathrm{lat}} + O(\alpha_s^2, a^2, 1/M_0^2, \alpha_s a/M_0). \tag{65}$$

Terms which grow linearly with aM_0 are no longer present in $\hat{C}^J_j$ and $\hat{Z}^J_{ij}$. Note that in the lattice NRQCD approach, discretization and relativistic corrections are intertwined since $O(a)$ and $O(1/M)$ interactions are treated as equally important.

VI. LARGE LOGARITHMS

Another feature of the $\hat{C}^J_j$ matching coefficients is the presence of $\ln(aM_0)$ terms. The simulations of heavy-light systems carried out to date have used bare heavy quark mass values in the range $aM_0 \sim 1.6 - 4.0$ where these logarithms are not large. However, in the large aM_0 limit, these logarithms must be treated with care. The logarithms appearing in the coefficients $\hat{C}^J_j$ for $j > 0$ are tamed by the $1/(aM_0)$ factors in their corresponding current operators. However, the logarithm appearing in the $\hat{C}^J_0$ coefficient becomes problematical and must be dealt with using the renormalization group.

In matching calculations between QCD and various continuum effective theories, one usually encounters similar logarithms of the form $\ln(M/\mu)$, where μ is some scale introduced by the renormalization procedure. Such logarithms are summed using the renormalization group (RG) equations which follow from the requirement that physical quantities must not depend on μ. Since μ appears only inside the logarithms, one ends up with simple anomalous-dimension matrices and RG equations which can be solved straightforwardly. The situation is more complicated in the present calculation. The role of μ is taken over by the inverse lattice spacing $1/a$, and a appears not only in the logarithms, but also in other places, such as the τ^J_j which are complicated functions of aM_0. Furthermore, the ultraviolet cutoff $1/a$ is an integral part of our effective theory and we cannot take $a \to 0$.

As discussed in Ref. [6], the observation that the left-hand side of Eq. (1) is independent of the lattice spacing can be exploited to derive an RG equation for the $\hat{C}^J_j$ coefficients. This equation describes the change in the $\hat{C}^J_j$ as the lattice spacing is varied. Collecting the $\hat{C}^J_j$ coefficients into a vector, the RG equation may be written

$$\left(a\frac{d}{da} + (\gamma^J)^{\mathrm{tr}} \right) \vec{\hat{C}}^J = 0, \tag{66}$$

where the anomalous dimension matrix is given by

$$\gamma^J_{ij}(\alpha_s, aM_0) = \sum_k \left(a\frac{d}{da}\hat{Z}^J_{ik} \right) (\hat{Z}^J)^{-1}_{kj}. \tag{67}$$

The right-hand side of Eq. (67) is a complicated function of aM_0. Once γ^J_{ij} is determined numerically for a large range of aM_0 values, Eq. (66) can be solved by numerical methods. Our primary concern was the determination of the matching coefficients $\hat{C}^J_j$ for values of aM_0 relevant for simulations of B and D mesons, and for such values, RG improvement was not needed. Hence, we have not attempted to obtain the entire anomalous dimension matrices for our current operators.

VII. SUMMARY

In summary, the spatial components of the heavy-light axial-vector current A_k and all components of the heavy-light vector current V_μ were expressed in terms of lattice operators suitable for use in simulations of B and D mesons. In the lattice theory, the heavy quarks were treated using the NRQCD formulation, the light quarks were described by the tadpole-improved clover action, and the standard Wilson action was used for the gluons. The light quarks were treated as massless. The expansions were carried out to $O(1/M)$ by matching appropriate scattering amplitudes to one-loop order in perturbation theory. We found

$$J_\mu = \sum_{j=0}^{N_{J_\mu}-1} C_j^J \; J_{\mu,\mathrm{lat}}^{(j)} + O(\alpha_s^2, a^2, 1/M_0^2, \alpha_s a/M_0), \quad (68)$$

where $N_{A_k} = N_{V_k} = 5$ and $N_{V_0} = 3$. The lattice current operators are given by

$$J_{\mu,\mathrm{lat}}^{(0)}(x) = \bar{q}(x)\,\Gamma_\mu^J\, Q(x), \quad (69)$$

$$J_{\mu,\mathrm{lat}}^{(1)}(x) = \frac{-1}{2M_0}\bar{q}(x)\,\Gamma_\mu^J\, \boldsymbol{\gamma\cdot\nabla}\, Q(x), \quad (70)$$

$$J_{\mu,\mathrm{lat}}^{(2)}(x) = \frac{-1}{2M_0}\bar{q}(x)\, \boldsymbol{\gamma\cdot}\overleftarrow{\boldsymbol{\nabla}}\,\gamma_0\;\Gamma_\mu^J\, Q(x), \quad (71)$$

$$J_{k,\mathrm{lat}}^{(3)}(x) = \frac{-1}{2M_0}\bar{q}(x)\,\Gamma_0^J \nabla_k\, Q(x) \quad (72)$$

$$J_{k,\mathrm{lat}}^{(4)}(x) = \frac{1}{2M_0}\bar{q}(x)\,\overleftarrow{\nabla}_k \Gamma_0^J\, Q(x), \quad (73)$$

with $\Gamma_\mu^V = \gamma_\mu$ and $\Gamma_\mu^A = \gamma_5\gamma_\mu$, and the coefficients are

$$C_i^J = \begin{cases} 1 + \alpha_s\; \rho_i^J(aM_0) + O(\alpha_s^2), \;(i=0,1), \\ \alpha_s\; \rho_i^J(aM_0) + O(\alpha_s^2), \;(i \geq 2), \end{cases} \quad (74)$$

where values of the $\rho_i^J(aM_0)$ are listed in Tables IV-VI. The currents can also be expressed in terms of improved current operators $\hat{J}_{\mu,\mathrm{lat}}^{(j)}(x)$ as shown in Eq. (65).

This completes the matching calculation through $O(\alpha_s/M)$ and $O(a\,\alpha_s)$ for all components of the vector and axial-vector heavy-light currents. Our matching coefficients have already been applied in leptonic B and B^* meson decays to extract the f_{PS} and f_V decay constants [12–15]. They are also relevant for studies of $B \to \pi$ or ρ semileptonic decays. In this article, we presented results only for the simple NRQCD action of Eqs. (32)-(34). Matching coefficients for other NRQCD actions which have appeared in the literature and for different values of (aM_0, n) are also available. For example, Ref. [12] used a slightly different action, and an action with higher-order improvement terms was employed in Refs. [13,15]. In all cases, we find that in the useful range $1.0 \leq aM_0 \leq 10.0$, the one-loop coefficients exhibit only a mild dependence on aM_0 and do not become particularly large.

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TABLE I. Values of the coefficients $\tau_i^{V_k}$ corresponding to the spatial components of the vector current V_k for various values of the bare heavy-quark mass aM_0 and NRQCD stability parameter n. Uncertainties in the determinations of these parameters due to the use of Monte Carlo integration are included.

aM_0	n	$\tau_0^{V_k}$	$\tau_1^{V_k}$	$\tau_2^{V_k}$	$\tau_3^{V_k}$	$\tau_4^{V_k}$
10.0	1	0.8189(1)	−0.588(3)	−13.93(1)	−0.566(4)	−0.940(5)
7.0	1	0.7880(1)	−0.640(2)	−8.150(7)	−0.508(3)	−0.909(3)
4.0	1	0.7221(1)	−0.757(5)	−2.809(7)	−0.385(7)	−0.836(7)
4.0	2	0.7340(1)	−0.742(4)	−3.084(8)	−0.422(6)	−0.842(7)
3.5	2	0.7176(2)	−0.775(4)	−2.310(5)	−0.392(6)	−0.817(6)
3.0	2	0.6982(1)	−0.821(4)	−1.594(4)	−0.351(5)	−0.791(5)
2.7	2	0.6840(2)	−0.856(3)	−1.196(4)	−0.323(5)	−0.767(5)
2.5	2	0.6732(2)	−0.877(3)	−0.955(4)	−0.309(5)	−0.750(4)
2.0	2	0.6410(1)	−0.962(3)	−0.427(3)	−0.254(4)	−0.693(3)
1.7	2	0.6170(2)	−1.027(2)	−0.188(2)	−0.218(3)	−0.647(3)
1.6	2	0.6075(2)	−1.051(2)	−0.123(2)	−0.205(3)	−0.627(2)
1.2	3	0.5768(2)	−1.175(2)	−0.081(2)	−0.150(3)	−0.534(2)
1.0	4	0.5559(2)	−1.265(2)	−0.148(1)	−0.108(3)	−0.464(2)
0.8	5	0.5249(2)	−1.387(2)	−0.288(1)	−0.049(3)	−0.363(1)

TABLE II. Values of the coefficients $\tau_i^{A_k}$ corresponding to the spatial components of the axial-vector current A_k, similar to Table I.

aM_0	n	$\tau_0^{A_k}$	$\tau_1^{A_k}$	$\tau_2^{A_k}$	$\tau_3^{A_k}$	$\tau_4^{A_k}$
10.0	1	0.2634(1)	−1.098(4)	2.624(18)	0.611(6)	−0.7291(2)
7.0	1	0.2685(1)	−1.145(3)	2.137(12)	0.636(4)	−0.7017(2)
4.0	1	0.2817(1)	−1.254(2)	1.494(6)	0.691(2)	−0.6401(2)
4.0	2	0.2742(1)	−1.197(2)	1.475(6)	0.590(2)	−0.6407(2)
3.5	2	0.2766(1)	−1.225(2)	1.339(5)	0.595(2)	−0.6219(2)
3.0	2	0.2793(1)	−1.259(1)	1.168(4)	0.598(2)	−0.5977(2)
2.7	2	0.2812(1)	−1.284(1)	1.065(3)	0.598(2)	−0.5796(2)
2.5	2	0.2824(1)	−1.303(1)	0.985(3)	0.600(2)	−0.5654(2)
2.0	2	0.2853(1)	−1.365(1)	0.750(2)	0.599(2)	−0.5196(2)
1.7	2	0.2862(1)	−1.416(1)	0.582(2)	0.595(2)	−0.4816(2)
1.6	2	0.2860(1)	−1.436(1)	0.513(2)	0.591(2)	−0.4661(2)
1.2	3	0.2725(1)	−1.510(2)	0.188(1)	0.537(2)	−0.3823(2)
1.0	4	0.2598(1)	−1.567(2)	−0.025(1)	0.511(2)	−0.3190(2)
0.8	5	0.2407(1)	−1.658(2)	−0.291(1)	0.500(2)	−0.2290(2)

TABLE III. Values of the coefficients $\tau_i^{V_0}$ corresponding to the temporal component of the vector current V_0, similar to Table I.

aM_0	n	$\tau_0^{V_0}$	$\tau_1^{V_0}$	$\tau_2^{V_0}$
10.0	1	0.3549(2)	−0.541(8)	1.706(19)
7.0	1	0.3961(2)	−0.572(6)	1.339(12)
4.0	1	0.4954(2)	−0.643(3)	0.982(12)
4.0	2	0.4868(1)	−0.695(3)	0.969(6)
3.5	2	0.5161(1)	−0.718(3)	0.907(5)
3.0	2	0.5542(1)	−0.752(3)	0.844(4)
2.7	2	0.5832(2)	−0.774(2)	0.815(4)
2.5	2	0.6058(1)	−0.794(2)	0.792(3)
2.0	2	0.6795(2)	−0.853(2)	0.744(3)
1.7	2	0.7417(2)	−0.899(2)	0.718(2)
1.6	2	0.7666(2)	−0.920(2)	0.707(2)
1.2	3	0.8832(2)	−1.034(1)	0.684(1)
1.0	4	0.9674(2)	−1.109(1)	0.682(1)
0.8	5	1.0853(2)	−1.202(1)	0.696(1)

TABLE IV. Values of the coefficients $\rho_j^{V_k}$ defined in Eq. (51) corresponding to the spatial components of the vector current for various values of the bare heavy-quark mass aM_0 and NRQCD stability parameter n. Uncertainties in the determinations of these parameters due to the use of Monte Carlo integration are included.

aM_0	n	$\rho_0^{V_k}$	$\rho_1^{V_k}$	$\rho_2^{V_k}$	$\rho_3^{V_k}$	$\rho_4^{V_k}$
10.0	1	−0.5051(3)	−0.128(5)	16.968(11)	0.991(4)	0.069(5)
7.0	1	−0.5441(2)	−0.202(4)	10.733(7)	0.932(3)	0.190(3)
4.0	1	−0.5496(2)	−0.346(5)	4.680(7)	0.810(7)	0.354(7)
4.0	2	−0.5744(2)	−0.382(5)	4.955(8)	0.846(6)	0.360(7)
3.5	2	−0.5694(2)	−0.421(5)	4.011(5)	0.816(6)	0.392(6)
3.0	2	−0.5585(2)	−0.443(5)	3.099(4)	0.775(5)	0.431(5)
2.7	2	−0.5472(2)	−0.470(4)	2.566(4)	0.747(5)	0.451(5)
2.5	2	−0.5366(2)	−0.495(4)	2.228(4)	0.734(5)	0.467(4)
2.0	2	−0.4958(2)	−0.521(3)	1.415(3)	0.678(4)	0.505(3)
1.7	2	−0.4561(3)	−0.551(3)	0.970(2)	0.642(3)	0.528(3)
1.6	2	−0.4391(3)	−0.564(3)	0.828(2)	0.630(3)	0.534(2)
1.2	3	−0.3679(3)	−0.609(3)	0.419(2)	0.574(3)	0.563(2)
1.0	4	−0.3018(4)	−0.606(3)	0.254(1)	0.532(3)	0.570(2)
0.8	5	−0.1818(5)	−0.597(4)	0.110(1)	0.473(3)	0.564(1)

TABLE V. Values of the coefficients $\rho_j^{A_k}$ corresponding to the spatial components of the axial-vector current, similar to Table IV.

aM_0	n	$\rho_0^{A_k}$	$\rho_1^{A_k}$	$\rho_2^{A_k}$	$\rho_3^{A_k}$	$\rho_4^{A_k}$
10.0	1	0.0504(2)	0.383(6)	0.413(18)	−0.186(6)	−0.1420(2)
7.0	1	−0.0246(2)	0.303(4)	0.447(12)	−0.211(4)	−0.0180(2)
4.0	1	−0.1093(2)	0.151(3)	0.377(6)	−0.267(2)	0.1578(2)
4.0	2	−0.1146(2)	0.073(3)	0.397(6)	−0.166(2)	0.1584(2)
3.5	2	−0.1285(1)	0.028(3)	0.362(5)	−0.171(2)	0.1963(2)
3.0	2	−0.1396(1)	−0.005(3)	0.337(4)	−0.174(2)	0.2375(2)
2.7	2	−0.1444(2)	−0.043(3)	0.305(3)	−0.174(2)	0.2641(2)
2.5	2	−0.1458(2)	−0.070(3)	0.288(3)	−0.176(2)	0.2826(2)
2.0	2	−0.1401(2)	−0.117(3)	0.238(2)	−0.174(2)	0.3316(2)
1.7	2	−0.1253(2)	−0.161(3)	0.200(2)	−0.171(2)	0.3625(2)
1.6	2	−0.1177(2)	−0.180(3)	0.191(2)	−0.166(2)	0.3727(2)
1.2	3	−0.0636(3)	−0.275(3)	0.151(1)	−0.112(2)	0.4110(2)
1.0	4	−0.0058(3)	−0.304(3)	0.131(1)	−0.087(2)	0.4251(2)
0.8	5	0.1023(5)	−0.326(4)	0.113(1)	−0.076(2)	0.4298(2)

TABLE VI. Values of the coefficients $\rho_j^{V_0}$ corresponding to the temporal component of the vector current, similar to Table IV.

aM_0	n	$\rho_0^{V_0}$	$\rho_1^{V_0}$	$\rho_2^{V_0}$
10.0	1	0.1712(3)	−0.387(9)	−0.433(19)
7.0	1	0.0599(2)	−0.482(7)	−0.065(12)
4.0	1	−0.1107(3)	−0.672(4)	0.291(12)
4.0	2	−0.1150(2)	−0.642(5)	0.305(6)
3.5	2	−0.1557(2)	−0.691(4)	0.367(5)
3.0	2	−0.2023(2)	−0.725(4)	0.430(4)
2.7	2	−0.2341(2)	−0.765(3)	0.458(4)
2.5	2	−0.2570(2)	−0.791(3)	0.481(3)
2.0	2	−0.3221(2)	−0.842(3)	0.529(3)
1.7	2	−0.3686(3)	−0.891(3)	0.555(2)
1.6	2	−0.3861(3)	−0.907(3)	0.566(2)
1.2	3	−0.4621(3)	−0.963(3)	0.589(1)
1.0	4	−0.5012(4)	−0.974(3)	0.591(1)
0.8	5	−0.5300(5)	−0.994(3)	0.577(1)